Estimation of Mixed Venous CO_2 Tension and QRS Electrical Axis From Simple Mathematical Considerations

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Abstract : Simple mathematical formulations are introduced to obtain mixed venous carbon dioxide tension ($P\bar{v}CO_2$) and mean electrical axis of the ventricles (θ). A linear decrease in O_2 and an exponential increase in CO_2 concentrations of alveolar gas during rebreathing are shown to be essential for the estimation of $P\bar{v}CO_2$. The QRS electrical axis was usually obtained from a graphical analysis using Einthoven's triangle with peak height differences between the R and S waves. The θ value is calculated from a simple trigonometric function instead of the graphical analysis. These examples indicate that elementary mathematics is useful in physiology to make clear physiological meaning behind observed phenomena.

Key words : elementary mathematics, mixed venous CO_2 tension, QRS electrical axis

Introduction

For the evaluation of cardiac output with direct Fick's method, mixed venous CO_2 tension ($P\bar{v}co_2$) is necessary together with arterial CO_2 tension ($Paco_2$). $Paco_2$ can be measured directly by taking arterial blood. Alveolar Pco_2 (P_Aco_2) is also a good approximation of $Paco_2$. However, $P\bar{v}co_2$ cannot be measured easily because of the sampling difficulty of mixed venous blood. Indirect estimation of $P\bar{v}co_2$ has been studied by a rebreathing method (Defares, 1958 : Mochizuki *et al*., 1984 ; Uchida *et al*., 1986 ; Vanhees *et al*., 2000). A rebreathing system is generally composed of the lung and a bag containing the air. Neglecting a dead space compared with a tidal volume, we can approximate Pco_2 in the bag during rebreathing as P_Aco_2. Since the rebreathing system is a closed system, CO_2 is accumulated and P_Aco_2 rises. With the progress of the P_Aco_2 increase, diffusion of CO_2 from the mixed venous blood to alveoli is reduced, and finally the diffusion is stopped when $P\bar{v}co_2 = P_Aco_2$. Therefore, we can get an information on the blood ($P\bar{v}co_2$) from that on the gas (P_Aco_2) without taking the blood. In this report fundamental relations necessary to estimate $P\bar{v}co_2$ from the O_2 and CO_2 concentrations during rebreathing are shown.

The standard mean electrical axis of the ventricles (θ) is 59 degrees, which is changed markedly in certain pathological conditions (Guyton, 1976). The θ value was usually determined from a graphical analysis of the standard leads electrocardiogram (ECG) using Einthoven's triangle. Projections of differences in peak heights between the R and S waves on the axes of leads I and III schematically give the θ value. Instead of the graphical analysis, θ is here shown to be obtained from an elementary mathematical consideration.

Estimation of Pco_2 of the mixed venous blood

P_Aco_2 rises exponentially during rebreathing (Defares, 1958), while alveolar Po_2 (P_Ao_2) is decreased linearly (Mochizuki *et al*., 1984 ; Uchida

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et al., 1986). Such a contrast in O_2 and CO_2 concentrations during rebreathing is due to the differencthees in pressure gradient between the alveolar gas and the mixed venous blood. The pressure gradient of CO_2 is about one tenth of that of O_2, and the equilibration between $P_{A}co_2$ and $P\bar{v}co_2$ is attained within a contact time of the mixed venous blood with the alveolar gas. According to these observations, time dependence of the O_2 and CO_2 concentrations during rebreathing can be written as

$$F(t) = a - bt \qquad (1)$$

$$G(t) = G_\infty - (G_\infty - G_0)\exp(-kt), \qquad (2)$$

where $F(t)$ and $G(t)$ are time dependent concentrations of O_2 and CO_2 in a rebrething gas, and a, b and k are positive constants (Fig.1). G_0 and G_∞ are CO_2 concentrations at the start and the end of rebreathing, respectively. Time dependent alveolar O_2 and CO_2 pressures are given by

$$P_{A}o_2(t) = (P_B - 47)\,F(t), \qquad (3)$$

$$P_{A}co_2(t) = (P_B - 47)\,G(t) \qquad (4)$$

where P_B is a barometric pressure and 47 Torr is saturated water vapor pressure at 37 ℃. At the start of rebreathing ($t=0$), $P_{A}o_2(0)$ and $P_{A}co_2(0)$ are calculated from the initial alveolar O_2 and CO_2 concentrations (F_0 and G_0). At the end of the rebreathing ($t \to \infty$), the limiting $P_{A}co_2(t)$, which is equal to G_∞ corresponds to $P\bar{v}co_2$ because of the equilibration of CO_2 tension between the alveolar gas and the mixed venous blood.

Neglecting the lung volume change during rebreathing, we have $\dot{V}o_2$ and $\dot{V}co_2$ as follows:

$$\dot{V}o_2 = -V(dF(t)/dt) = Vb \qquad (5)$$

$$\dot{V}co_2 = V(dG(t)/dt) = Vk(G_\infty - G_0)\exp(-kt) \qquad (6)$$

Equation (5) shows that $\dot{V}o_2$ remains constant even in rebreathing, and Eq. (6) shows that $\dot{V}co_2$ is decreased exponentially with the time. From Eqs. (5) and (6), respiratory quotient (RQ) is given by

$$RQ = (k/b)\,(G_\infty - G_0)\exp(-kt), \qquad (7)$$

which is rewritten with Eq. (2) as

$$RQ = (k/b)\,\{G_\infty - G(t)\}. \qquad (8)$$

Substituting $G(t)$ in Eq. (4) for that in Eq. (8), we have

$$RQ = -(k/b)\,P_{A}co_2(t)/(P_B - 47) + (k/b)G_\infty. \qquad (9)$$

Decreasing $\dot{V}co_2$ and constant $\dot{V}o_2$ during rebreathing give rise to a linear reduction of RQ against the $P_{A}co_2(t)$ increase (Eq. (9)). The limiting value G_∞ is equal to the $G(t)$ value when $RQ = 0$, reflecting the fact that at this stage no CO_2 output occurs because of the disappearance of pressure gradient between alveoli and mixed venous blood. Therefore,

$$P\bar{v}co_2 = (P_B - 47)\,G_\infty = (P_B - 47)\,G(t)_{RQ=0}. \qquad (10)$$

Mochizuki *et al*. (1984) developed a method to obtain $P\bar{v}co_2$ from a linear relation between $P_{A}co_2$ and RQ during rebreathing. The above discussion shows that their method is based on the fundamental two relations given by Eqs. (1) and (2).

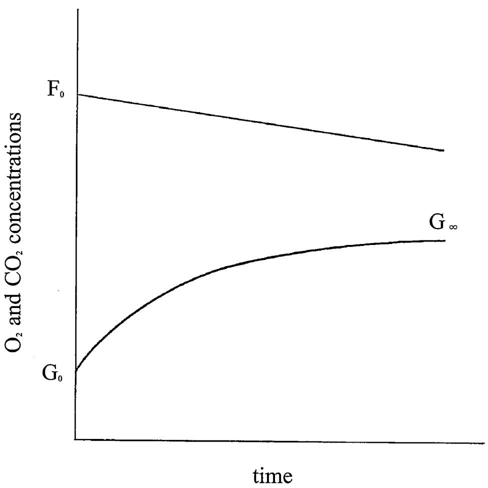

Fig. 1 Changes in O_2 and CO_2 concentrations of alveolar gas during rebreathing

Estimation of the QRS electrical axis

The QRS electrical axis (θ) of ECG was usually obtained using the differences in peak heights between R and S waves for the leads I and Ⅲ (Fig.2). In this figure, O is the middle point of Einthoven's triangle, and A is an intersection point with the projections of leads I and Ⅲ. Segments OB and AC correspond to the peak height differences between the R and S waves for the leads I and Ⅲ, and are represented here **I** and **Ⅲ**, respectively. The θ value is obtained by measuring the angle $\angle BOA$. It should be noted that $\angle BOC = \angle BAC = 30°$, and therefore $\angle OAC = \angle OAB + \angle BAC = (90° - \theta) + 30° = 120° - \theta$. Referring to the two triangles ΔOAB and ΔOAC in this figure, we have

$$OA\cos\theta = OB = \mathbf{I} \qquad (11)$$

$$OA\cos(120° - \theta) = AC = \mathbf{Ⅲ}. \qquad (12)$$

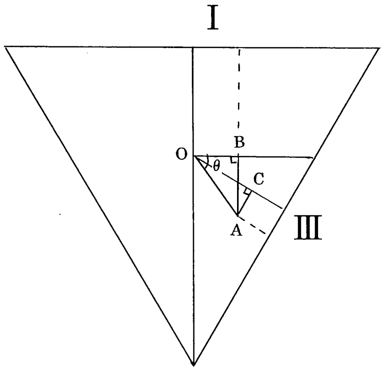

Fig. 2 Relation between the QRS electrical axis and peak heights of the standard leads ECG

From Eqs. (11) and (12)

$$(-1/2 \cdot \cos\theta + \sqrt{3}/2 \cdot \sin\theta) / \cos\theta = \mathbf{III} / \mathbf{I} \quad (13)$$

Therefore,

$$\tan\theta = 2/\sqrt{3}\,(\mathbf{III} / \mathbf{I} + 1/2)\,. \quad (14)$$

The QRS electrical axis θ can be calculated as follows without drawing a diagram like Fig. 2:

$$\theta = \tan^{-1}\{2/\sqrt{3}\,(\mathbf{III} / \mathbf{I} + 1/2)\}\,. \quad (15)$$

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